Laborator 5

REZOLVAREA SISTEMELOR DE ECUAŢII LINIARE

Aplicaţia 1.

Solve the following system of linear equations.

$$\begin{cases} 0.3x_1+0.52x_2+x_3=-0.01 \\ 0.5x_1+x_2+1.9x_3=0.67 \\ 0.1x_1+0.3x_2+0.5x_3=-0.44 \end{cases}$$

a) by Cramer's rule

b) using the commands specific to the Matlab programs

$$A=\begin{bmatrix} 0,3 & 0,52 & 1 \\ 0,5 & 1 & 1,9 \\ 0,1 & 0,3 & 0,5 \end{bmatrix}$$

Find the determinant:

$$\Delta = \begin{vmatrix} 0,3 & 0,52 & 1 \\ 0,5 & 1 & 1,9 \\ 0,1 & 0,3 & 0,5 \end{vmatrix} = 0,3 \times 1 \times 0,5 - 0,3 \times 1,9 \times 0,3 - 0,52 \times (0,5 \times 0,5 - 1,9 \times 0,1) +$$

$$+0,52 \times 1 \times (0,5 \times 0,3 - 1 \times 0,1) = -0,0022$$

$$\Delta_1 = \begin{vmatrix} -0,01 & 0,52 & 1 \\ 0,67 & 1 & 1,9 \\ -0,44 & 0,3 & 0,5 \end{vmatrix} = -0,01 \times (1 \times 0,5 - 0,3 \times 1,9) - 0,52 \times (0,67 \times 0,5 + 0,44 \times 1,9) +$$

$$+1 \times (0,67 \times 0,3 + 1 \times 0,44) = 0,03278$$

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{0,03278}{-0,0022} = -14,9$$

$$x_2 = \frac{\Delta_2}{\Delta} = \frac{0,0649}{-0,0022} = -29,5$$

$$x_3 = \frac{\Delta_3}{\Delta} = \frac{-0,04356}{-0,0022} = 19,8$$

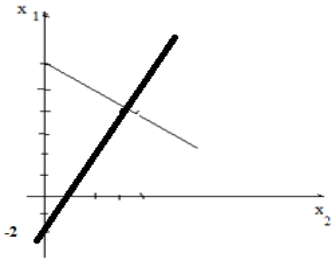
Aplicaţia 2.

Solve graphically and by eliminating the unknowns**:**

$$\begin{cases} 3x_1 + 2x_2 = 18 \\ -x_1 + 2x_2 = 2 \end{cases}$$

$$x_1 = -\left(\frac{2}{3}\right) \cdot x_2 + 6$$

$$x_1 = -\left(\frac{2}{3}\right) \cdot x_2 + 6$$



The solution of the system is given by the coordinates of the intersection point of the two straight lines

$x_1 = 4$

$x_2 = 3$

Verificare:

$34 + 23 = 18$

$-4 + 23 = 2$

Aplicaţia 3.

Solve the system of equations by eliminating Gaussian

$$\begin{cases} 3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 = -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 = 71.4 \end{cases}$$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 3 & -0.1 & -0.2 & 7.85 \\ 0.1 & 7 & -0.3 & -19.3 \\ 0.3 & -0.2 & 10 & 71.4 \end{array}\right]$$

1. elimination

2. Substitution

1.The goal of elimination is to transform the coefficient matrix into a higher triangular matrix

$$\left[\begin{array}{ccc|c} 3 & -0.1 & -0.2 & 7.85 \\ 0.1 & 7 & -0.3 & -19.3 \\ 0.3 & -0.2 & 10 & 71.4 \end{array}\right]$$

Divide Equation 1 through 3 and multiply by 0.1

Subtract from the equation 2

$$\begin{array}{l} \left[\begin{array}{cccc} 0.1 & -1 & -0.2/0.1 & \vdots \quad 7.85/0.1 \end{array}\right] \\ -\left[\begin{array}{cccc} 0.1 & 7 & -0.3 & \vdots \quad -19.3 \end{array}\right] \\ \hline \left[\begin{array}{cccc} 0 & 7.00333 & -0.293333 & \vdots \quad -19.5617 \end{array}\right] \end{array}$$

The following results are obtained:

$$\left[\begin{array}{ccc|c} 3 & -0.1 & -0.2 & 7.85 \\ 0 & 7.00333 & -0.293333 & -19.5617 \\ 0.3 & -0.2 & 10 & 71.4 \end{array}\right]$$

Divide Equation 1 through 3 and multiply by 0.3

Subtract from Equation 3

$$\begin{array}{l} \left[\begin{array}{cccc} 0.3 & -0.1 \cdot 0.3/3 & -0.2 \cdot 0.3/3 & \vdots \quad 7.85 \cdot 0.3/3 \end{array}\right] \\ -\left[\begin{array}{cccc} 0.3 & -0.2 & 10 & \vdots \quad 71.4 \end{array}\right] \\ \hline \left[\begin{array}{cccc} 0 & 0.19 & -10.02 & \vdots \quad -70.615 \end{array}\right] \end{array}$$

The result is

$$\begin{bmatrix} 3 & -0.1 & -0.2 & \vdots & 7.85 \\ 0 & 7.00333 & -0.293333 & \vdots & -19.5617 \\ 0 & 0.19 & -10.02 & \vdots & -70.615 \end{bmatrix}$$

Divide Equation 2 by 7.00333 and multiply by -0.2

Subtract from Equation 3

$$\begin{array}{r} \begin{bmatrix} 0 & -0.2 & -4.76 & \vdots & 335.968 \end{bmatrix} \\ -\begin{bmatrix} 0 & -0.2 & -5.46 & \vdots & -336.728 \end{bmatrix} \\ \hline \begin{bmatrix} 0 & 0 & 0.7 & \vdots & 0.76 \end{bmatrix} \end{array}$$

The result:

$$\begin{bmatrix} 3 & -0.1 & -0.2 & \vdots & 7.85 \\ 0 & 7.00333 & -0.293333 & \vdots & -19.5617 \\ 0 & 0 & 10.0120 & \vdots & 70.0843 \end{bmatrix}$$

1. Substitution

$$\begin{bmatrix} 3 & -0.1 & -0.2 & \vdots & 7.85 \\ 0 & 7.00333 & -0.293333 & \vdots & -19.5617 \\ 0 & 0 & 10.0120 & \vdots & 70.0843 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -0,1 & -0,2 \\ 0 & 7,00333 & -0,293333 \\ 0 & 0 & 10,0120 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19,5617 \\ 70,0843 \end{bmatrix}$$

$$10.0120a_3 = 70.0843$$

$$a_3 = \frac{70.0843}{10.0120}$$

$$a_3 = 7.00003$$

$$\begin{bmatrix} 3 & -0,1 & -0,2 \\ 0 & 7,00333 & -0,293333 \\ 0 & 0 & 10,0120 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19,5617 \\ 70,0843 \end{bmatrix}$$

$$7.00333a_2 - 0.293333a_3 = -19.5617$$

$$a_2 = \frac{0.293333a_3 - 19.5617}{7.00333}$$

$$a_2 = -2.50000$$

$$\begin{bmatrix} 3 & -0,1 & -0,2 \\ 0 & 7,00333 & -0,293333 \\ 0 & 0 & 10,0120 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19,5617 \\ 70,0843 \end{bmatrix}$$

$$3a_1 - 0.1a_2 - 0.2a_3 = 70.0843$$

$$a_1 = \frac{70.0843 + 0.1a_2 + 0.2a_3}{3}$$

$$= 3.00000$$

PROGRAME MATLAB

- **Rezolvarea sistemelor de ecuaţii liniare prin metode simple**

```
% TO BE RESOLVED BY THE CRAMER RULE:
%0.3 x1 + 0.52 x2 +     x3=-0.01
%0.5 x1 +      x2 + 1.9 x3= 0.67
%0.1 x1 + 0.30 x2 + 0.5 x3=-0.44

The matrix of coefficients and the matrix associated with% each unknown is constructed
A=[....]
A1=
A2=
A3=

x1=det(A1)/det(A)
x2=det(A2)/det(A)
x3=det(A3)/det(A)

% RESOLVE GRAPHIC AND BY THE ELIMINATION OF UNKNOWN
% 3 x1 + 2 x2 = 18
%  -x1 + 2 x2 = 2

x1=-10:10;
x2=
xx2=

plot(x1,x2,x1,xx2)
grid
zoom

% TO RESOLVE THE SYSTEM. OF EQUATIONS FOR THE ELIMINATION OF GAUSS-IANĂ
%   3 x1 - 0.1 x2 - 0.2 x3 = 7.85
% 0.1 x1 +   7 x2 - 0.3 x3 = -19.3
% 0.3 x1 - 0.2 x2 +  10 x3 = 71.4

l1=[3 -0.1 -0.2 7.85];
l2=[0.1 7 -0.3 -19.3];
l3=[0.3 -0.2 10 71.4];
AC=[l1; l2; l3];
%pause

l2=l2-l1*l2(1)/l1(1);
l3=l3-l1*l3(1)/l1(1);
AC=[l1;l2;l3]
%pause

l3=l3-l2*l3(2)/l2(2);
AC=[l1;l2;l3]
%pause

x3=l3(4)/l3(3)
x2=(l2(4)-l2(3)*x3)/l2(2)
x1=(l1(4)-l1(3)*x3-l1(2)*x2)/l1(1)

% RESOLVING EQUATION SYSTEMS (DEFECTUOASE!)
%     x1 + 2 x2 = 10       si          x1 + 2 x2 = 10
```

% 1.1 x1 + 2 x2 = 10.4 1.05 x1 + 2 x2 = 10.4

Homework

Given the following set of equations:

$$\begin{cases} x_1 + 7x_2 - 3x_3 = -47 \\ 4x_1 - 4x_2 + 8x_3 = 64 \\ 12x_1 - x_2 + 3x_3 = 8 \end{cases}$$

To calculate the solutions of this system using Gauss elimination