

## Laboratoire 6

### Application 1

Considering the equation :

$$\begin{cases} 11x_1 - 9x_2 = 99 \\ 11x_1 + 13x_2 = 286 \end{cases}$$

Solve this system of equations by the Gauss-Seidel method, for the equations in this order in the system and respectively for the case when the order is reversed. To discuss the convergence of the method.

Solution:

$$\begin{aligned} x_1 &= 99/11 + 9/11 * x_2; \\ x_2 &= 286/13 - 11/13 * x_1; \\ x_1 &= 0; \quad x_2 = 0; \end{aligned}$$

### Iteratia 1

$$x_2 = 0$$

$$x_1 = 99/11 = 9$$

$$x_2 = 286/13 - (11/13) * 9 = 14.384$$

### Iteratia 2

$$x_2 = 14.384$$

$$x_1 = 99/11 + (9/11) * 14.384 = 20.768$$

$$x_2 = 286/13 - (11/13) * 20.768 = 4.427$$

$$\varepsilon_i = \left| \frac{20.768 - 9}{20.768} \right| 100 = 56.66\%$$

### Application 2

To calculate using the Gauss-Seidel method, without relaxation ( $\lambda = 1$ ) and with relaxation ( $\lambda = 0.9$ ), and with a maximum allowable error of  $5e^{-5}$ , the next equation system.:

$$\begin{cases} -5x_1 + 12x_3 = -60 \\ 4x_1 - x_2 + x_3 = -2 \\ 6x_1 + 8x_2 = 45 \end{cases}$$

**Indicație:** It is observed that the last equation has the element on the diagonal equal to 0, so the last 2 equations will be pivoted. For A(i,i) to be major equations 1 and 3 are pivoted again

$$\begin{cases} 4x_1 - x_2 + x_3 = -2 \\ 6x_1 + 8x_2 = 45 \\ -5x_1 + 12x_3 = -60 \end{cases}$$

**Aleg**  $x_1 = x_2 = x_3 = 0$

$$x_1 = \frac{-2 + x_2 - x_3}{4}$$

$$x_2 = \frac{45 - 6x_1}{8}$$

$$x_3 = \frac{-60 + 5x_1}{12}$$

### Iteration 1

$$x_2 = x_3 = 0$$

$$x_1 = \frac{-2 + 0 - 0}{4} = -0.5$$

$$x_1 = -0.5$$

$$x_3 = 0$$

$$x_2 = \frac{45 - 6(-0.5)}{8} = 6$$

$$x_1 = -0.5$$

$$x_3 = 6$$

$$x_3 = \frac{-60 + 5(-0.5)}{12} = -5.208$$

### Iteration 2

$$x_2 = 6$$

$$x_3 = -5.208$$

$$x_1 = \frac{-2 + 6 + 5.208}{4} = 2.302$$

$$\varepsilon_r = \left| \frac{2.302 - x_r}{x_r} \right| 100 = \dots$$

$$\varepsilon_1 = \left| \frac{2.302 - (-0.5)}{2.302} \right| 100 = 121.72\%$$

$$x_1 = 2.302$$

$$x_3 = -5.208$$

$$x_2 = \frac{45 - 6(2.302)}{8} = 3.8985$$

$$x_1 = 2.302$$

$$x_2 = 3.8985$$

$$x_3 = \frac{-60 + 5 \cdot 2.302}{12} = -4.040$$

Improving the convergence by using relaxation can be done as follows: after iterating each unknown, it is recalculated using a relaxation factor  $\lambda$  :

$$x_i^{nou} = \lambda x_i^{nou} + (1 - \lambda) x_i^{vechi}$$

- If  $0 \leq \lambda \leq 1$ , the calculation is with sub-relaxation - used to make a non-convergent system convergent
- If  $1 \leq \lambda \leq 2$  : the calculation is over-relaxation - used to accelerate the convergence of an already converged system

## Solving systems of linear equations by iterative methods

### % CONVERGENCE CRITERION FOR THE GAUSS-SEIDEL METHOD

```
clear all
clf
% Se consideră ecuațiile:
%  $11x_1 - 9x_2 = 99$ 
%  $11x_1 + 13x_2 = 286$ 

x1=-5:30;
x2=286/13-11/13*x1;
x22=-99/9+11/9*x1;
subplot(2,1,1); plot(x1,x2,'r',x1,x22,'g'); grid; hold on
subplot(2,1,2); plot(x1,x2,'r',x1,x22,'g'); grid; hold on

x1=0; x2=0;
subplot(2,1,1); plot(x1,x2,'co')
subplot(2,1,2); plot(x1,x2,'c*')

disp('Pornește iterația')
pause
N=0;
epsi=1;
while abs(epsi)>0.01
    x1old=x1; x2old=x2;
    N=N+1
    x1=99/11+9/11*x2;
    x2=286/13-11/13*x1;
        subplot(2,1,1)
        plot(x1,x2,'o')
    [x1 x2];
    epsi1=abs(x1-x1old)/x1; epsi2=abs(x2-x2old)/x2;
    epsi=max(epsi1,epsi2);
    pause
end
[x1 x2]

disp('Reverse the order of equations and press a key')
pause
x1=0; x2=0;
epsi=1
N=0
while abs(epsi)>0.01
    x1old=x1; x2old=x2;
    N=N+1
    x1=286/11-13/11*x2;
    x2=-99/9+11/9*x1;
        subplot(2,1,2)
        plot(x1,x2,'*')
    [x1 x2];
    epsi1=abs(x1-x1old)/x1; epsi2=abs(x2-x2old)/x2;
    epsi=max(epsi1,epsi2);
    disp('Press Ctrl + C to finish the program')
    pause
end
```

## % RELAXATION METHOD FOR THE GAUSS-SEIDEL ALGORITHM

% It is considered a system of 3 linear equations for which:

A=[-5 0 12;4 -1 1;6 8 0];

C=[-60;-2;45];

% Note that the last equation has the element on the diagonal = 0

% The last two equations are pivoted:

A=[-5 0 12;6 8 0; 4 -1 1];

C=[-60;45;-2];

% The equations such that A (i, i) are major again are pivoted

A=[4 -1 1;6 8 0;-5 0 12];

C=[-2;45;-60];

X=[0;0;0];

% the proposed value for starting the iteration

er\_adm=5e-5;

% admissible error

epsilon=1;

% it starts with some value > er\_adm

% without relaxation: lambda = 1

disp('== Rezolvarea sistemului fără relaxare ==')

N=0;

while epsilon > er\_adm,

XX=X;

%

X\_vechi

X(1)=(C(1)-A(1,2)\*X(2)-A(1,3)\*X(3))/A(1,1);

X(2)=(C(2)-A(2,1)\*X(1)-A(2,3)\*X(3))/A(2,2);

X(3)=(C(3)-A(3,1)\*X(1)-A(3,2)\*X(2))/A(3,3);

epsilon=max((X-XX)./X);

N=N+1;

end

N

X

epsilon

% with sub-relaxation: lambda < 1

disp('=== Rezolvarea sistemului cu relaxare ===')

disp('press any key to continue the program')

pause

la=0.9; X=[0;0;0]; epsilon=1;

N=0;

while epsilon > er\_adm,

XX=X;

%

X\_vechi

X(1)=(C(1)-A(1,2)\*X(2)-A(1,3)\*X(3))/A(1,1);

X(1)=la\*X(1)+(1-la)\*XX(1);

X(2)=(C(2)-A(2,1)\*X(1)-A(2,3)\*X(3))/A(2,2);

X(2)=la\*X(2)+(1-la)\*XX(2);

X(3)=(C(3)-A(3,1)\*X(1)-A(3,2)\*X(2))/A(3,3);

X(3)=la\*X(3)+(1-la)\*XX(3);

epsilon=max((X-XX)./X);

N=N+1;

end

N

X

epsilon

### Homework

Solve the following system of equations

$$\begin{cases} 4x_1 - 2x_2 - x_3 = 40 \\ x_1 - 6x_2 + 2x_3 = -28 \\ x_1 - 2x_2 + 12x_3 = -86 \end{cases}$$

using a) metod Gauss-Seidel

b) metod Jacobi

and an admisibile error of 5%

Solve on paper, 2 iteration,  
not octave. make photo and  
add to pdf