1) Méthode Bisection

$f(x) = x^2 - 12x + 11 = 0 \quad , \quad x \in [-2, 5]$

Iteration 1 : $x \in [-2, 5]$

$f(-2) = 39$

$f(5) = 25 - 60 + 11 = -24$

$f(-2) \cdot f(5) < 0 \Rightarrow \quad x_1 = \frac{-2+5}{2} = 1,5$

$|\varepsilon| = \left| \frac{x_i - x_{i+1}}{x_i} \right| \cdot 100 = \left| \frac{a-b}{b+a} \right| \cdot 100 = \left| \frac{-2-5}{-2+5} \right| \cdot 100 = 233,33\,\%$

Iteration 2 : $x \in [1,5 \, ; \, 5]$

$f(1,5) = 2,25 - 18 + 11 = -4,75 < 0$

$f(1,5) \cdot f(5) > 0 \Rightarrow \quad x_2 = \frac{1,5+5}{2} = 3,25$

$|\varepsilon| = \left| \frac{5+1,5}{5-1,5} \right| \cdot 100 = 185\,\%$

$|\varepsilon_r| = \left| \frac{3,25-1,5}{3,25} \right| \cdot 100 = 53,8\,\%$

Iteration 3 : $x \in [1,5 \, ; 3,25]$

$f(3,25) = -17,43$

$f(1,5) \cdot f(3,25) > 0 \Rightarrow x_3 = \frac{1,5+3,25}{2} = 2,375$

$|\varepsilon| = \left| \frac{3,25+1,5}{3,25-1,5} \right| \cdot 100 = 271\,\%$

$|\varepsilon_r| = \left| \frac{2,375-3,25}{2,375} \right| \cdot 100 = 36\,\%$

Iteration 4 : $x \in [\quad ; \quad]$

$f(-1{,}125) = 26{,}1$

$f(-1{,}125) \cdot f(-0{,}25) > 0 \Rightarrow x_4 = \dfrac{-1{,}125 - 0{,}25}{2} = -0{,}68$

$|\varepsilon| = \left| \dfrac{-1{,}125 + 0{,}25}{-1{,}25 - 0{,}25} \right| \cdot 100 = 58\ \%$

$|\varepsilon_r| = \left| \dfrac{-0{,}68 + 1{,}125}{-0{,}68} \right| \cdot 100 = 65\ \%$

2) Méthode de Jacobi

$x_1 = 1,\ x_2 = 0\ ,\ x_3 = 1$

$$\begin{cases} 12x_1 + 3x_2 - 5x_3 = 1 \\ 1x_1 + 5x_2 + 3x_3 = 28 \\ 3x_1 + 7x_2 + 13x_3 = 76 \end{cases} \Rightarrow \begin{cases} x_1 = 1 - 3x_2 + 5x_3 \\ x_2 = \dfrac{28 - x_1 - 3x_3}{5} \\ x_3 = \dfrac{76 - 3x_1 - 7x_2}{13} \end{cases}$$

Iteration 1 :

$x_2 = 0\ ;\ x_3 = 1 \Rightarrow \boxed{x_1 = 0{,}5}$

$x_1 = 1\ ;\ x_3 = 1 \Rightarrow x_2 = \dfrac{28 - 1 - 3}{5} = 4{,}8 \Rightarrow \boxed{x_2 = 4{,}8}$

$x_2 = 0\ ;\ x_1 = 1 \Rightarrow x_3 = \dfrac{76 - 3}{13} = 24{,}33 \Rightarrow \boxed{x_3 = 24{,}33}$

Iteration 2 :

$x_1 = \dfrac{1 - 3(4{,}8) + 5(24{,}33)}{12} = 9{,}02 \qquad \varepsilon_1 = \left| \dfrac{9{,}02 - 0{,}5}{9{,}02} \right| \cdot 100 = 94{,}45\%$

$x_2 = \dfrac{28 - 0{,}5 - 3(24{,}33)}{5} = -9{,}098 \qquad \varepsilon_2 = \left| \dfrac{9{,}02 - 4{,}8}{9{,}02} \right| \cdot 100 = 46{,}78\%$

$x_3 = \dfrac{76 - 3(0{,}5) - 7(4{,}8)}{13} = 3{,}14 \qquad \varepsilon_3 = \left| \dfrac{-9{,}098 - 24{,}83}{-9{,}098} \right| \cdot 100 = 367{,}42\%$

3) L'interpolation linéaire

$t = 16\,s$

$t(s)$	0	10	15	20	22	30
$v(m/s)$	0	220	360	510	600	900

$$f(x) = a_0 + a_1 x$$

$$S_r = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

$$v(t) = v(t_0) + \frac{v(t_1) - v(t_0)}{t_1 - t_0}(t - t_0)$$

$t_0 = 15$; $v(t_0) = 360$

$t_1 = 20$; $v(t_1) = 510$

$$v(16) = v(15) + \frac{510 - 360}{20 - 15}(16 - 15) =$$

$$= 360 + \frac{510 - 360}{20 - 15}(16 - 15) =$$

$$= 360 + \frac{150}{5} \cdot 1 =$$

$$= 360 + 30 =$$

$$= 390 \quad \Rightarrow \quad \underline{t = 16 \;,\; v(t) = 390}$$